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HEAT AND MASS TRANSFER IN THE PROCESS OF FUEL COMBUSTION IN AN AIR STREAM

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Abstract—Equations are obtained on the basis of a proposed model of mass transfer and combustion of a flow of fuel in a turbulent air stream, which describe the laws of combustion of gaseous and liquid fuels. The influence of pressure and of coefficients of air excess and fuel consumption on the rate of combustion is determined. The results obtained are compared with experimental data.

NOMENCLATURE

a.	characteristic dimension of a cascade (nozzle);
c,	oxygen concentration;
c_k,	concentration of the component k;
c_p,	heat capacity at constant pressure;
D,	coefficient of molecular diffusion;
E,	body force;
G_s,	component of gravity;
G_k,	total mass flow of the given component;
J_k,	diffusion flow of the component k;
J_q,	heat flow due to heat conduction and radiation;
k'	$= f(T, v)$, total constant of the reaction rate per unit volume which reveals the influence of kinetics and mass transfer;
l,	turbulence scale;
l_c.	characteristic linear dimension of a combustion chamber;
m,	variable mass of an element of fuel;
p,	gas pressure;
Q,	heat of combustion of fuel;
q,	chemical reaction rate per unit volume;
q_n,	rate of heat release from the reaction;
q_v,	heat dissipation due to friction and diffusion;
R,	characteristic dimension of a mole;
R_1,	gas constant;
R_s,	reaction force;
r,	radius of a liquid fuel droplet;
S,	reaction surface per unit volume;
S_1,	entropy of a mass unit;
T,	flow temperature;
u_s,	velocity component of a fuel element in the direction of motion;
u',	pulsation velocity;
V,	volume of one mole;
v,	velocity in the given point of space at the given moment of time;
v_k,	velocity of the component k;
W_s,	force of action of a gas flow on an element of fuel in a given direction;
Nu,	Nusselt number;
Pr,	Prandtl number;
Re,	Reynolds number.

Greek symbols

$\bar{\alpha}$,	total coefficient of heat transfer;
α_h,	total coefficient of heat transfer between a particle and the surrounding medium;
β,	coefficient of total mass transfer;
γ_l,	specific weight of liquid fuel;
η,	viscosity coefficient;
θ_w,	surface temperature of particle;
ν_k.	stoichiometric coefficient times the molecular weight of the given component;
ρ,	gas density;
$\bar{\rho}$,	evaporation heat of a liquid fuel;
ω,	angular rotation velocity of fuel moles.

COMBUSTION of a fuel in furnaces and ovens should be considered in terms of a real system open to the action of the surrounding medium.

A number of physical and chemical processes take place in this system which are accompanied by mass motion, energy transport and entropy change.

A closed system of equations describing these processes consists of:

(1) the equation of gas motion (motion of a viscous compressible fluid)

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla p + \bar{F} + \eta \Delta \mathbf{v} + \left(\xi + \frac{\eta}{3}\right) \nabla \operatorname{div} \mathbf{v}. \quad (1)$$

(2) The equation of motion of fuel particles:

$$W_s = m \frac{\mathrm{d}u_s}{\mathrm{d}t} + u_s \frac{\mathrm{d}m}{\mathrm{d}t} + G_s - R_s. \quad (2)$$

(3) The equation of continuity:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{v}) = 0. \quad (3)$$

(4) The equation of continuity for the component k (allowing for chemical reaction):

$$\frac{\partial c_k}{\partial t} = -\operatorname{div} G_k + \nu_k q. \quad (4)$$

(5) The kinetic equation of a reaction:

$$q = k' f(G_k, c, S). \quad (5)$$

(6) The equation of state of the gas:

$$p = \rho g R_1 T. \quad (6)$$

(7) The equation of energy interchange:

$$\rho T \frac{\mathrm{d}S_1}{\mathrm{d}t} = -\operatorname{div} J_q + q + q_\nu. \quad (7)$$

The equation of evaporation of liquid fuel particles should be introduced into the system of equations in case of the combustion of a flow of a liquid fuel

$$\gamma_l \bar{\rho} \frac{\mathrm{d}r}{\mathrm{d}t} = \alpha_h (T - \theta_w). \quad (8)$$

Here:

∇p,	pressure gradient;
η, ξ,	coefficients of viscosity;
$(\rho \mathbf{v}) = \sum_1^n c_k \mathbf{v}_k$,	total flow of all the mixture components;
$G_k = c_k v + J_k$,	total mass flow of the given component;
$J_k = D \nabla c_k$,	diffusion flow of the component k.

The system of equations is solved for given initial and boundary conditions and, in particular, at the initial temperature distribution and boundary conditions of heat transfer between the gas flow and the surrounding medium. The system includes eight independent variables: ρ, p, v, u, r, c, c_k, T.

The kinetic equation in this system expresses the connexion between the rate of the fuel burn-up, acting concentrations of oxygen and fuel per unit volume and main physico-chemical factors (kinetics and mass transfer) on which the reaction rate depends.

When dealing with a hydraulic one-dimensional problem the following approximate kinetic equation of the reaction rate per unit volume can be used in all the cases of combustion of gaseous, liquid or solid fuel:

$$-G_{f_0} \frac{\mathrm{d}\Omega}{\mathrm{d}x} = k' c c' \quad (9)$$

where

c',	fuel concentration per unit volume of the reaction space;
G_{f_0},	initial consumption of a fuel per unit time per unit flow cross-sectional area;
G_f,	fuel consumption per unit time per unit flow cross-sectional area at the distance x from the initial section;
$\Omega = \frac{G_f}{G_{f_0}}$,	relative amount of unburnt fuel at the distance x from the initial section;

Evidently, $1 - \Omega$ is the degree of fuel burn-up. Consider the combustion of a gaseous fuel. If we imagine an ideal preliminary mixing of the gaseous fuel with the atmospheric oxygen then the combustion process of such a mixture will be wholly homogeneous. In this case the reaction proceeds over the whole volume and $k' = k$, i.e. is equal to the reaction rate constant which varies with temperature by Arrhenius' law.

If the mixing of oxygen with inflammable gas takes place during the process of combustion, then the gas enters the combustion zone in the form of volumes not yet mixed with oxygen (in the form of "moles").

The burn-up of gas moles takes place over their surface of contact with the surrounding atmospheric oxygen.

The atmospheric oxygen is transported to this surface by turbulent mass transfer and penetrates inside the molar volumes.

The depth of penetration of the reaction inside the volume δ is determined simultaneously by molecular diffusion and by reaction kinetics.

The value of the surface of contact between a fuel and oxygen can be expressed per unit volume of the reaction space S in the following way:

$$S = \frac{(1-m)s}{V} \tag{10}$$

where

s,	surface;
V,	volume of one mole;
$1-m$,	total volume of moles per unit volume of the reaction space.

Characteristic linear dimensions of a mole

$$R = \sqrt[3]{V} \text{ and } l_s = \sqrt{s} \qquad [6]$$

enter a constant relationship

$$l_s/R = E_s^{1/2} = \text{const.}$$

and

$$s = \frac{E_s(1-m)}{R}. \tag{11}$$

The mean initial dimension of a mole R_0 depends on the boundary conditions of atomisation of the initial unmixed volumes of the fuel, namely on:

(1) the relative dimension a/l_c of the entry device (cascades or nozzles) through which the gas passes, where a is the characteristic dimension of the cascade, l_c is the characteristic linear dimension of the combustion chamber;

(2) the geometric characteristic of the cascade t_s/b and β_s where t_s is the cascade pitch, b is the cascade width and β_s is the angle of incidence of the cascade profile. This dependence may be written in the form

$$\frac{R_0}{a} = f_1\left(\frac{a}{l_c}, \frac{t_s}{b}, \beta_s\right). \tag{12}$$

As the distance from the initial section of the cascade increases the characteristic dimension R of a mole will change depending on mean (along the section) hydrodynamic characteristics of the flow at the given relative distance x/l_c namely, on the degree of turbulence l and on the relative velocity of the turbulent motion superimposed on the mean velocity v of the main flow.

The degree of turbulence l of such motion is of the size of the cascade, and the relative velocity of the turbulent motion at rather a large distance from the cascade is purely pulsation velocity u'.

The degree of atomisation of the initial fuel volumes depends on the value of the Reynolds number characterizing the influence of the pulsation velocity u' and on the scale of turbulence l, $Re_l = (u'l)/\nu$ when motion is highly turbulent, and is accompanied by considerable pulsation velocities. Large-scale pulsations play a predominant part in turbulent flow and their velocity is proportional to a mean flow velocity in the given section whilst the scale of turbulence is proportional to the characteristic dimension of the initial turbulating device (entry cascade or nozzle) "a" and, moreover, it is the function of the relative distance up to the section considered

$$x/l_c \text{ or } x/a.$$

According to [1]

$$l \sim a(x/a)^{2/7}. \tag{13}$$

Hence, it follows that the characteristic dimension of a mole R may be expressed in terms of the number $Re = (v \,.\, l_c)/\nu$ (proportional to Re_l) and of a/l_c.

Thus, in the end, the influence of the hydrodynamics on the current mole dimension R is expressed by a relationship of the form f_2 $(Re, a/l_c, x/l_c)$ which reflects the change of the initial dimension of a mole as a function of Re and relative distance x/l_c.

As we shall see later, the influence of turbulence or vorticity in the flow results in additional atomisation of moles at the expense of the increase in the Re of the main flow.

Furthermore, the fuel burn-up influences the dimension of a mole of the fuel whilst it moves with the flow.

When the influence of the hydrodynamics of the flow is absent $R/R_0 = \Omega^{1/3}$, and in the general case, the relation for the current characteristic dimension of a mole of reacting gaseous fuel entering the chamber through the given cascade can be expressed as follows:

$$\frac{R}{a} = \sqrt[3]{\Omega}\,.\,f_2\left(Re,\ \frac{a}{l_c},\ \frac{x}{l_c}\right). \qquad (14)$$

Thus, the dimensions of the unmixed combustible volumes in the flow of air, and, consequently, their total surface per unit volume are the functions of the relative distance and the relative characteristic linear dimensions of the combustion chamber and entry device, the Reynolds number and of the fuel degree of burn-up.

The burn-up of the fuel in the air flow takes place as a result of the interaction between chemical kinetics and mass transfer.

In the kinetic equation (9) the interaction between these factors is taken into account by the introduction of the total constant of the reaction rate k', the expression of which can be obtained from the following considerations:

The total fuel burn-up rate at the interface between a fuel volume and oxygen is determined by equality between the diffusion oxygen flow into the reaction zone and the amount of oxygen expended in chemical reaction.

Substituting the coefficient of the total mass transfer $\beta = (Nu\,.\,D)/l_c$ (l_c is the characteristic linear dimension of the combustion chamber) we obtain the reaction rate per unit boundary surface:

$$k_s = \beta(c - c_w) = k c_w \gamma_f \delta \qquad (15)$$

where

Nu,	diffusion Nusselt number;
c_w,	oxygen concentration on the surface of a fuel mole;
γ_f,	specific weight or fuel concentration (per unit volume of a fuel mole);
$k = k_0 \exp(-E/R_1T)$,	kinetic constant of chemical reaction rate;
$\delta = \sqrt{\left(\frac{D}{k\gamma_f}\right)}$,	effective depth of penetration of the reaction in a fuel mole;
E,	activation energy of the chemical reaction.

On the other hand, from equation (9) the reaction rate per unit volume is

$$q = k'cc' \qquad (9a)$$

c_w may be eliminated from equation (15). In such a case

$$k_s = c/(1/\beta + 1/k\gamma_f\delta) \qquad (16)$$

The reaction rate per unit volume is

$$q = k_s\,.\,S = k'cc'. \qquad (17)$$

Substituting into (17) the expression for S from (11), as well as $c' = \gamma_f\,.\,(1 - m)$ and k_s from (16) and eliminating c it is easy to find the following expression for the total constant of the reaction rate per unit volume:

$$k' = \frac{E_s}{\gamma_f R}\Big/(1/\beta + 1/k\gamma_f\delta). \qquad (18)$$

Equation (18) enables three ultimate regimes of a combustion process to be considered:

(1) For

$$\beta \to \infty;\quad k' = \frac{E_s\,.\,k\delta}{R} \qquad (19)$$

an internal diffusive-kinetic regime will prevail at which the total fuel combustion rate is determined both by the rate of molecular diffusion of oxygen in the reaction zone at the depth δ and by the rate of the chemical reaction between the fuel and oxygen.

(2) For

$\beta \to \infty$ and

$$\delta = \frac{R}{E_s} = \frac{v}{s}\,;\ k' = k = k_0 \exp(-E/RT_1) \qquad (20)$$

the combustion transforms into the kinetic regime over the whole volume of fuel moles where fuel combustion rate is determined only by the chemical reaction rate.

(3) For

$$k = \infty;\quad k' = \frac{E_s\,.\,\beta}{R\gamma_f} \qquad (21)$$

there will be a purely diffusive regime, when the fuel combustion rate is determined only by the rate of oxygen supply to the reacting surface of fuel moles.

The rate of oxygen supply to the reaction zone depends on the intensity of mass transfer to the surface of fuel moles in a turbulent flow.

The intensity of mass transfer in the given cross-section of a combustion chamber is expressed by the known number $Nu = (\beta l_c/D)$ and depends on the pulsation velocity of the turbulent motion and on the scale of turbulence l.

Hence it follows that the criterion Nu as well as the relative dimension of a mole can be made to depend on the Reynolds criterion $Re = vl_c)/\nu$, the relative characteristic dimension of the entry turbulating device a/l_c and on the relative distance x/l_c and also on the geometric characteristic of the entry cascade (nozzle).

It is known moreover, from the general theory of heat and mass transfer, that Nu depends also on the criterion $Pr = \nu/D$, i.e. on the diffusion Prandtl number which expresses the relation between the physical constants: kinematic viscosity coefficient ν and the coefficient of molecular diffusion D.

As a result of all these considerations given above, the value Nu (for the given entry device) is expressed by some function

$$Nu = \frac{\beta l_c}{D} = f_3\left(Re, \frac{a}{l_c}, \frac{x}{l_c}, Pr\right). \tag{22}$$

Applying the general conclusions from the theory of fuel flow combustion [2, 3] and assuming the velocity of moles motion u equal to the gas velocity v we can find the value of the gaseous fuel concentration per unit volume of the reaction space in the following form:

$$c' = \gamma_f(1 - m) = c_0 \frac{M}{\alpha} \Omega \cdot \frac{v_0}{v} \tag{23}$$

where

M,	stoichiometric coefficient of reaction;
α,	coefficient of excess oxygen;
T_0, c_0,	initial temperature of flow and oxygen concentration;
v_0, v,	gas velocities at the initial and given flow cross-sections.

The value of the current oxygen concentration c can be obtained in the form

$$c = c_0 \left[1 - \frac{1}{\alpha}(1 - \Omega)\right] \frac{v_0}{v} \tag{24}$$

by simultaneous solutions of the general continuity equation and the equation of continuity for oxygen (taking account of the reaction).

Substituting the values of c and c' into the kinetic equation (9) we get

$$- G_{f_0} \frac{d\Omega}{dx} = k' c_0^2 \frac{M}{\alpha} \Omega \left[1 - \frac{1}{\alpha}(1 - \Omega)\right] \left(\frac{v_0}{v}\right)^2 \tag{25}$$

or assuming that

$$v \simeq v_0 \frac{T}{T_0}$$

we have

$$- G_{f_0} \frac{d\Omega}{dx} = k' c_0^2 \frac{M}{\alpha} \Omega \left[1 - \frac{1}{\alpha}(1 - \Omega)\right] \left(\frac{T_0}{T}\right)^2 \tag{26}$$

The process of combustion of a liquid fuel in an air flow proceeds in a similar manner, inasmuch as the burning atmospheric oxygen reacts with fuel vapour, however, this process has some peculiarities.

The flow of a liquid fuel spray represents a non-uniform system consisting of liquid particles of the fuel, its vapour, atmospheric oxygen reacting with them and combustion products.

The atomised liquid forms fuel volumes just as in the case of combustion of gaseous fuel without preliminary mixing with air. The characteristic dimension of a mole R is also a function of both boundary conditions of the fuel supply to the combustion chamber, the relative distance from the initial section of the chamber, the Reynolds number Re and of the degree of the fuel burn-up.

However, in contrast with the burn-up of gaseous fuel volumes the volumes of a liquid fuel in an air flow include, in general both vapour and liquid phases.

As the liquid phase of the fuel volumes is the source of vaporization in the process of fuel burn-up, the molar dimensions R and the mean weight concentration of the fuel vapour c' in any given section depend not only on the intensity of the fuel burn-up but also on the rate of evaporation of liquid particles.

The fuel vapour concentration c' in the kinetic equation for the rate of combustion of a liquid fuel can be obtained in the following form, from the continuity equation of the flow consisting of both gas, of vapour and liquid phases of the fuel allowing for burn-up

$$c' = c_0 \frac{M}{a}\left[\Omega - \frac{\gamma_l (1 - m_0) v_0 \vartheta^3}{G_{f_0}}\right]\frac{v_0}{v} \tag{27}$$

assuming that the mean velocity of the flow in any given section is equal to the mean velocity of the fuel volumes in this same section [3].

where

γ_l,	specific weight of the liquid fuel;
ϑ^3,	ratio of volumes of liquid fuel particles in a given section, and the initial section, i.e. degree of evaporation;
$1 - m_0$,	total volume of fuel moles in a unit volume of the reaction space in the initial section.

If we neglect the evaporation in a preparatory zone the initial vapour concentration is

$$c'_0 = 0; \quad \gamma_l v_0 (1 - m_0) = G_{f_0}$$

and

$$c' = c_0 \frac{M}{a}[\Omega - \vartheta^3]\frac{v_0}{v}. \tag{28}$$

A mean characteristic molar dimension of burning liquid fuel, which is in fact a mixture of liquid and vapour phase, can be found from the continuity equation written down for a fuel flow allowing for burn-up and liquid phase evaporation, as follows:

$$\Omega - \vartheta^3 \cdot \left(1 - \frac{\gamma_f}{\gamma_l}\right) = \frac{\gamma_f}{\gamma_l}\left(\frac{R}{r_0}\right)^3 \tag{29}$$

where

γ_f,	specific weight of vapour;
γ_l,	specific weight of liquid fuel phase;
r_0,	initial radius of droplet of liquid fuel (in the present paper we do not consider polydispersion and distribution of particles of various dimensions as it has no significance in this context).

The initial radius of a droplet can be related to the characteristic relative dimension of the nozzle atomising the fuel and to the parameters determining the dispersion of atomisation.

By neglecting γ_f/γ_l in comparison with 1 we obtain equation (29) in a simpler form:

$$\Omega - \vartheta^3 = \frac{\gamma_f}{\gamma_l} \cdot \left(\frac{R}{r_0}\right)^3. \tag{30}$$

Hence

$$R = r_0 \sqrt[3]{\left[\frac{\gamma_l}{\gamma_f}(\Omega - \vartheta^3)\right]}. \tag{31}$$

Hydrodynamic factors influence the characteristic dimension of a mole R just as in the case of gaseous fuel combustion. Taking these factors into account we get

$$\frac{R}{r_0} = \sqrt[3]{\left[\frac{\gamma_l}{\gamma_f}(\Omega - \vartheta^3)\right]} \cdot f_4\left(Re, \frac{a}{l_c}, \frac{x}{l_c}\right). \tag{32}$$

In the given case a is a characteristic linear dimension of an air-directing device determining the initial atomisation of fuel moles when the liquid fuel vapour is mixed with air entering through the cascade of the air-directing device.

Substituting c' and c into kinetic equation (9) we obtain the equation of the combustion of atomised liquid fuel in an air flow:

$$-G_{f_0}\frac{d\Omega}{dx} = k' c_0^2 \frac{M}{a}\left[1 - \frac{1}{a}(1 - \Omega)\right](\Omega - \vartheta^3)\left(\frac{v_0}{v}\right)^2. \tag{33}$$

Equation (32) is more general than (26) and transforms into the latter at $\vartheta^3 = 0$.

It is seen from (32) that in case of combustion of a liquid fuel the degree of the fuel burn-up in a considered section x depends not only on the vapour combustion rate, but also on the evaporation rate of the particles of the liquid fuel.

From the equation of liquid fuel evaporation (8) it is easy to determine the evaporation criteria:

$$p_1 = \frac{a_f T x}{\gamma_l \bar{\rho} u r_0} = \frac{a_f T_0 x}{\gamma_l \bar{\rho} u_0 r_0}. \tag{34}$$

From the kinetic equation the criterion of the burn-up is as follows:

$$p_2 = \frac{k' c_2^0 x}{G_{f_0} \alpha} \left(\frac{u_0}{u}\right)^2. \tag{35}$$

It is the relation of these criteria which influences the dependence of the degree of combustion on the degree of evaporation. With large ratios of p_2/p_1, $\Omega \cong 0$ at $\vartheta^3 = 0$. This means that in the limiting case when the vapour combustion rate is very high as compared with evaporation rate, the rate of combustion of a liquid fuel is determined by the evaporation rate.

Conversely, at low ratios of p_2/p_1, $\Omega \cong 1{\cdot}0$ at $\vartheta^3 = 0$ which means that in the limiting case when the vapour combustion rate is small as compared to the evaporation rate, then the combustion rate of a liquid fuel is equal to the vapour rate, and up to the moment of fuel ignition no liquid phase will be present in the fuel volumes.

The analysis of the experimental material shows that in several cases of combustion of a liquid fuel in an air flow one may approximately assume that $\Omega - \vartheta^3 = \text{const.}$, where Ω is always higher than ϑ^3, and then kinetic equation (33) will take the following form (at $v = v_0\, T/T_0$):

$$G_{f_0} \frac{\mathrm{d}\Omega}{\mathrm{d}x} = k' c_0^2 \frac{M}{\alpha} \left[1 - \frac{1}{\alpha}(1 - \Omega)\right] \left(\frac{T_0}{T}\right)^2 = \text{const.} \tag{36}$$

The combustion of a fuel under complicated aerodynamic flow conditions and, in particular, in a turbulent stream is of a special interest when analysing laws of combustion of gaseous and liquid fuel in an air flow.

It is known from numerous experiments with combustion of fuels in a turbulent stream that the turbulence of air at the entry into the combustion chamber allows the higher velocities of flow and high thermal fluxes.

The increase in the intensity of the fuel burn-up in a vortex flow is due to both the improvement in thermal conditions of the process at its initial stage and to the considerable step up of mass transfer in the reaction zone. Basically, the increase is accounted for by the fact that the turbulence of the flow gives rise to a sharp increase in local velocities where the combustion of a fuel proceeds. As a result of this there will be an increase in the intensity of mass transfer since the coefficient of mass transfer β is directly proportional to the Reynolds number.

Moreover, when the Reynolds number increases in a gas flow, then additional atomisation of fuel moles takes place and extension of their surfaces per unit volume of the reaction space occurs because the vorticity causes the circulation of fuel moles moving in an air flow.

Assume a rotation of fuel moles with angular velocity ω. Applying the law of conservation of moment of momentum we get

$$\Sigma m_1 R^2 \omega = \text{const.} \tag{37}$$

where R is the characteristic dimension of a mole, Σm_1 is the sum of masses of all the fuel moles in the given section; which remains constant (without considering burn-up).

It follows from (37) that the characteristic dimension of a fuel mole decreases and, accordingly, the surface S (11) increases in proportion to $\sqrt{\omega}$ or $v^{0{\cdot}5}$, because the angular velocity of rotation in a similar vortex flow is proportional to its average (mean) velocity v.

Experimental data on combustion in the vortex flow give a very close relation between intensification of the process and v as a result of the turbulence [4].

When the velocity of oxygen supply to the molar surface is the determining factor of the combustion rate, we may use equation (21) and simplify the kinetic equations both for a gaseous, and liquid, fuel.

According to (14, 22 and 32) the values β and R entering this equation are functions of the same parameters Re, x/l_c, a/l_c representing the influence of boundary conditions, hydrodynamics and the relative distance from the initial section. As in heat transfer we may express them as power relationships.

To solve the kinetic equations it is necessary to know the dependence of flow temperature in a given section on the degree of the fuel burn-up, heat transfer with the surrounding medium and on the intensity of heat transfer along the flow.

A relation can be obtained by a simultaneous solution of energy and continuity equations [2], and the expression for temperature can be approximately written in the following form:

$$T = \left[T_0 + \frac{G_{f0}Q'}{\gamma_0 v_0 c_p}(1 - \Omega)\right] \exp - \frac{\bar{a} \,.\, x}{\gamma_0 v_0 c_p R_1} \quad (38)$$

where

$$Q' = Q - \frac{a c_p \gamma_0 (T_0 - T_1)}{M \,.\, c_0}$$

is thermal effect of the reaction minus the heat expended in preheating the mixture up to ignition temperature T_0;

Q,	heat of fuel combustion;
T_1,	temperature of an inflammable mixture of a fuel and air;
c_p,	heat capacity;
γ_0, T_0, v_0,	specific weight, temperature and flow velocity in the initial section $x = 0$;
$\bar{a}$,	total heat transfer coefficient equal to the sum of $\bar{a}_{\text{radiant}} + \bar{a}_{\text{convective}}$.

By using relation (38) it is possible to solve the kinetic equation of burn-up, obtain the laws of fuel combustion in a flow and determine qualitatively the influence of various parameters on the intensity of the fuel burn-up both in a non-vortex turbulent and vortex flows in combustion chambers.

The relations derived from the solution of the kinetic fuel combustion equations describing the combustion process of gaseous and liquid fuel in an air stream, and the combustion of liquid fuel in a vortex (turbulent) stream of air, were checked by experiment.

The experiments were carried out by burning a liquid fuel (paraffin oil and Diesel fuel) in cylindrical cooled combustion chambers 214 and 185 mm in diameter under various conditions of fuel and air supply.

Laws of the fuel burn-up in terms of the length of a chamber in turbulent stream were investigated at fuel consumptions $G_f = 19{\cdot}7$–42 kg/h at coefficients of air excess $\alpha = 1{\cdot}0$–$1{\cdot}82$, at pressures $p = 1$–$5{\cdot}0$ kg/cm^2 and at thermal fluxes up to 35×10^6 kcal/m^3 h per unit volume of the combustion zone.

The results determined the constant coefficients in the burn-up equation and revealed the quantitative relations of the laws of fuel burn-up in simplified form: for the combustion chamber $d = 214$ mm

$$-\ln\left[1 - \frac{1 - \Omega}{\alpha}\right] = 5{\cdot}2\,\frac{x^{0{\cdot}6}}{\alpha}\,\frac{1}{B^{0{\cdot}2}}\left(\frac{1 - \Omega}{\alpha}\right)^{0{\cdot}75} \quad (39)$$

for the combustion chamber $d = 185$ mm

$$-\ln\left[1 - \frac{1 - \Omega}{\alpha}\right] = 5{\cdot}8\,\frac{x^{0{\cdot}52}}{\alpha}\,\frac{1}{B^{0{\cdot}2}}\left(\frac{1 - \Omega}{\alpha}\right)^{0{\cdot}75}. \quad (40)$$

Here

$$B = \frac{G_{f0} \,.\, \pi \,.\, D_c^2}{4} \cdot 3600 \text{ kg/h}$$

D_c is the diameter of the combustion chamber.

The results of experiments given in Figs. 1 and 2 are in good agreement with the curves plotted from equations (39) and (40) over a wide range of variations of the fuel consumption B and of the coefficient of air excess α.

The experimental investigations of the process of combustion of a liquid fuel were carried out in cylindrical combustion chambers with various turbulators on Diesel fuel (in a combustion chamber with $d = 214$) and on ethyl alcohol (in a laboratory combustion chamber with $d = 65$).

The experiments were carried out in the combustion chamber with $d = 214$ mm at the same

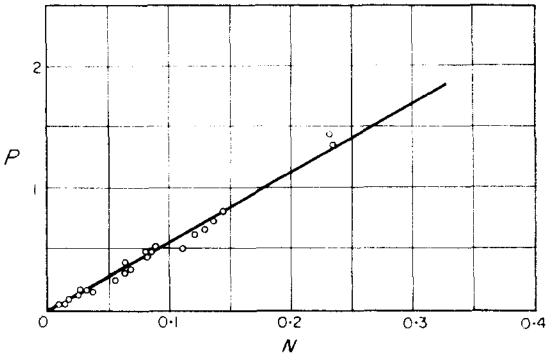

FIG. 1. Relation between fuel burn-up degree $(1 - \Omega)$ along the length of the flame x and B and α in a straight-through turbulent flow in a combustion chamber with $d = 214$ mm

$$P = -\ln\left(1 - \frac{1 - \Omega}{\alpha}\right)$$

$$N = 5{\cdot}2\,\frac{x^{0{\cdot}6}}{\alpha}\,\frac{1}{B^{0{\cdot}2}}\left(\frac{1 - \Omega}{\alpha}\right)^{0{\cdot}75}.$$

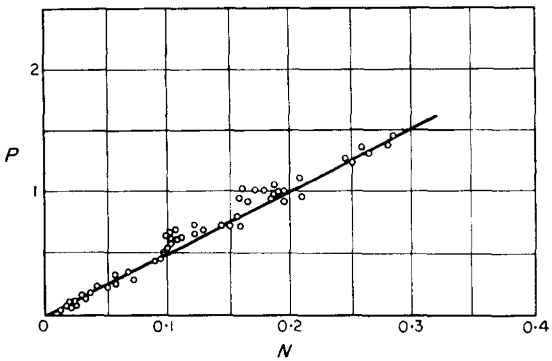

FIG. 2. Relation between the fuel burn-up degree $(1 - \Omega)$ along the length of flame x and B and α in a straight-through turbulent flow in a combustion chamber with $d = 185$ mm

$$P = -\ln\left(1 - \frac{1-\Omega}{\alpha}\right)$$

$$N = 5{\cdot}8\,\frac{x^{0\cdot 52}}{\alpha}\,\frac{1}{B^{0\cdot 2}}\left(\frac{1-\Omega}{\alpha}\right)^{0\cdot 75}.$$

parameters as in the case of the investigation of combustion in a straight-through turbulent flow and in the combustion chamber with $d = 65$ mm at fuel consumptions $B = 1{\cdot}0$–$8{\cdot}2$ kg/h, at coefficients of air excess $\alpha = 1{\cdot}0$–$1{\cdot}25$ and pressures $p = 1{\cdot}0$–$7{\cdot}25$ kg/cm^2.

On substituting the constant coefficients obtained from the experiment into equations of the liquid fuel burn-up in a vortex flow, they acquire the following form:

(a) for the combustion chamber with $d = 65$ mm

$$-\ln\left[1 - \frac{1}{\alpha}(1-\Omega)\right] = 6{\cdot}05\,\frac{x^{0\cdot 6}}{\alpha}\left(\frac{1-\Omega}{\alpha}\right)^{1\cdot 2\times 0\cdot 65}\left(\frac{B\,.\,\alpha}{p}\right)^{0\cdot 2} \tag{41}$$

and for the combustion chamber with $d = 214$ mm

$$-\ln\left[1 - \frac{1}{\alpha}(1-\Omega)\right] = 3{\cdot}18\,\frac{x^{0\cdot 6}}{\alpha}\left(\frac{1-\Omega}{\alpha}\right)^{1\cdot 2\times 0\cdot 75}\left(\frac{B\,.\,\alpha}{p}\right)^{0\cdot 2}. \tag{42}$$

Comparison between experimental results and curves plotted from equations (41) and (42) are given in Figs. 3 and 4.

The solution of the equation of the gaseous fuel burn-up in an air flow accounting for non-isothermal flow along the chamber length can be written in the form:

$$F\left(\frac{1-\Omega}{\alpha}, \alpha\right) = A_3\,\frac{\alpha^{0\cdot 8 - 0\cdot 45\,.\,m_3}}{G_{f_0}^{0\cdot 2}}\cdot\frac{x^{0\cdot 8 - 0\cdot 45\,.\,m_4}}{0{\cdot}8 - 0{\cdot}45\,.\,m_4} \tag{43}$$

where

$$P = F\left(\frac{1-\Omega}{\alpha}, \alpha\right) \equiv \int_0^{1-\Omega} \frac{\mathrm{d}\Omega}{\left[1 - \dfrac{1-\Omega}{\alpha}\right]\left(\dfrac{1-\Omega}{\alpha}\right)^{n_2\times 0\cdot 45}\cdot\Omega^{2/3}}. \tag{44}$$

In equation (43) the constant coefficients have been obtained by analysing experimental data [5] on the combustion of natural gas from the

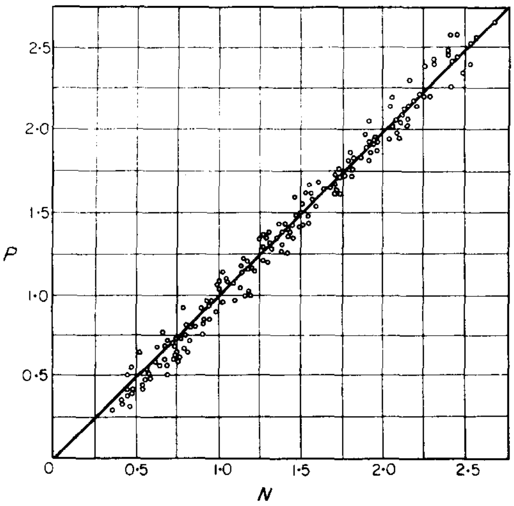

FIG. 3. Relation between fuel burn-up degree $(1 - \Omega)$ along the length of the flame x and B, p and α in a vortex turbulent flow in the combustion chamber with $d = 65$ mm

$$P = -\ln\left(1 - \frac{1-\Omega}{\alpha}\right)$$

$$N = 6{\cdot}05\,\frac{x^{0\cdot 6}}{\alpha}\left(\frac{1-\Omega}{\alpha}\right)^{1\cdot 2\times 0\cdot 65}\left(\frac{B\,.\,\alpha}{p}\right)^{0\cdot 2}.$$

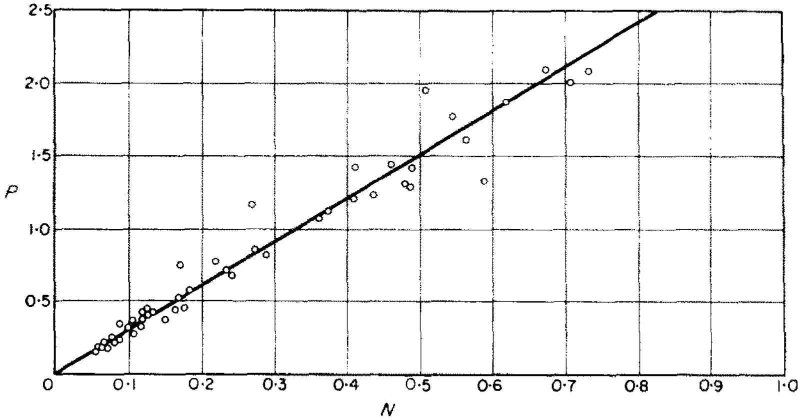

FIG. 4. Relation between the fuel burn-up $(1 - \Omega)$ along the length of the fuel flame x and B, p, α in a vortex flow in a cylindrical combustion chamber with $d = 214$ mm

$$P = -\ln\left(1 - \frac{1 - \Omega}{\alpha}\right)$$

$$N = 3{\cdot}18\,\frac{x^{0\cdot6}}{\alpha}\left(\frac{1 - \Omega}{\alpha}\right)^{1\cdot2\times0\cdot75}\left(\frac{B\alpha}{p}\right)^{0\cdot2}.$$

Dashava deposits in a vertical cylindrical combustion chamber 325 mm in diameter without swirling in air.

In the experiments the analysis of combustion products was carried out in six sections over the length of the chamber at fuel consumptions from 0·73 up to 3·0 kg/h.

In accordance with the conditions of the burn-up of a gas flame in such a combustion chamber the kinetic equation has the form:

$$P = F\left(\frac{1 - \Omega}{\alpha}, \alpha\right) = 3{\cdot}2\,\frac{\alpha^{0\cdot6}\,x^{0\cdot6}}{B^{0\cdot2}}. \qquad (45)$$

A comparison between the experimental data and a curve plotted from equation (44) is given in Fig. 5. The comparison between the laws of the burn-up of a liquid and those of gaseous fuel in a turbulent flow shows that the influence of the regime parameters (α, B) on the intensity of the combustion process is similar.

Increase in the fuel consumption decreases the intensity of the burn-up, increase in the coefficient of oxygen excess up to certain optimum limits leads to the increase in this intensity, and then the intensity falls sharply with the increase in α. Under the conditions considered (mixing during the combustion process) the pressure hardly influences the intensity of combustion of a gas flame. The combustion of a gas flame and of atomised liquid fuel is determined in most

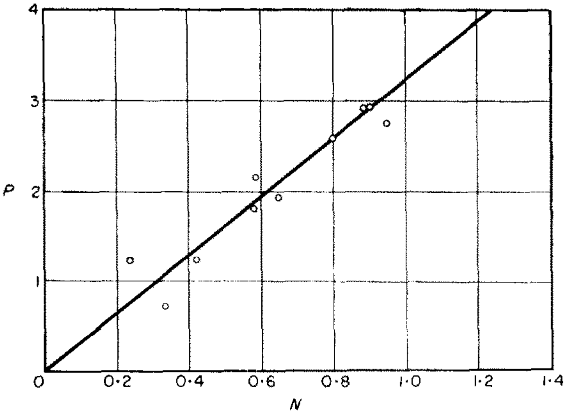

FIG. 5. Relation between the fuel burn-up degree $(1 - \Omega)$ along the length of the fuel flame x and the regime parameters of combustion chamber performance with $d = 325$ mm at combustion of a gaseous fuel

$$P = F\left(\frac{1 - \Omega}{\alpha}, \alpha\right)$$

$$N = \frac{\alpha^{0\cdot6}\,x^{0\cdot6}}{B^{0\cdot2}}.$$

cases not by the chemical reaction rate, but by the mass transfer in the motion of fuel in an oxidant flow. Thus, the scheme of mass transfer in the process of fuel combustion adopted in our analysis rightly depicts the actual processes of fuel combustion in an air flow.

CONCLUSIONS

(1) The role of mass transfer between a fuel and atmospheric oxygen during the process of fuel combustion in an air flow is considered.

(2) The model of mass exchange and combustion of a fuel flow in a turbulent air flow is proposed based on the interrelation between the fuel volumes and the air flow surrounding them. The fuel volumes intermix with the air flow and burn-up over the surface to a depth depending both on the kinetics and on the diffusion of oxygen into the interior of the volume.

(3) The theoretical analysis of the laws of combustion of a fuel flow under non-isothermal conditions is carried out, regard being paid to the kinetics and mass exchange during the process of combustion.

(4) The equations describing the burn-up laws of a fuel (liquid and gaseous) in a turbulent flow are obtained, and the influence both of the regime pressure parameters, the coefficient of oxygen excess and the fuel consumption on the combustion rate is determined.

Conclusions obtained are compared with experimental data.

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Аннотация—На основании предложенной модели массообмена и горения потока воздуха получены уравнения, описывающие закономерность выгорания газообразных и жидких топлив и определено влияние давления, коэффициента избытка воздуха и расхода топлива на скорость горения. Полученные выводы сопоставлены с экспериментальными данными.

Zusammenfassung—An Hand eines Modells für Stofftransport und Verbrennung eines Brennstoffstrahls im turbulenten Luftstrom wurden Gleichungen für die Verbrennung gasförmiger und flüssiger Brennstoffe ermittelt. Der Einfluss des Druckes, der Luftüberschusszahl und des Brennstoffverbrauches auf die Verbrennungsgeschwindigkeit wurde bestimmt. Die Ergebnisse wurden mit Versuchsdaten verglichen.

Résumé—Les équations décrivant les lois de la combustion des fuels liquides ou gazeux sont obtenues en partant d'un modèle déterminé de combustion et de transport de masse d'un écoulement de fuel dans un courant d'air turbulent. On a déterminé l'influence de la pression et des coefficients d'excès d'air et de consommation du fuel sur le taux de combustion. Les résultats obtenus sont comparés aux données expérimentales.